Angel and Shreiner: Interactive Computer Graphics, Eighth Edition

Chapter 11 Odd Solutions

11.1 $(m+1)^3$

11.3 As u varies over (a,b), $v = \frac{u-a}{b-a}$ varies over $(0,1)$. Substituting into the polynomial $p(u) = \sum_{k=0}^{n} c_k u^k$, we have
$q(v) = \sum_{i=0}^{v} d_i v^i = \sum_{k=0}^{n} c_k((b-a)v+a)^k$. We can expand the products on the right and match powers of v to obtain $\{d_i\}$.

11.5 Consider the Bernstein polynomial

$$b_{kd}(u) = \begin{pmatrix} d \\ k \end{pmatrix} u^k(1-u)^{d-k}.$$

For $k=0$ or $k=d$, the maximum value of 1 is at one end of the interval (0,1) and the minimum is at the other because all the zeros are at 1 or 0. For other values of k, the polynomial is 0 at both ends of the interval and we can differentiate to find that the maximum is at $u = k/d$. Substituting into the polynomial, the maximum value is $\frac{d!}{d^d}\frac{k^k}{k!}\frac{(d-k)^{d-k}}{(d-k)!}$ which is always between 0 and 1.

11.7 Any quadric can be written as

$$q(x,y,z) = ax^2+by^2+cz^2+2dxy+2exz+2fyz+2gx+2hy+2iz+j=0,$$

where a,b,c,d,e,f,g,h,i and j are constants. Let $\mathbf{p}^T = \begin{bmatrix} x & y & z & 1 \end{bmatrix}$. Then, we can rewrite the equation as

$$q(\mathbf{p}) = \mathbf{p}^T\mathbf{Q}\mathbf{p} = 0,$$

where

$$\mathbf{Q} = \begin{bmatrix} a & d & e & g \\ d & b & f & h \\ e & f & c & i \\ g & h & i & j \end{bmatrix}.$$

Note that we can also use $\mathbf{p}^T = \begin{bmatrix} x & y & z & w \end{bmatrix}$ where w can be any constant.

11.15 For $r=0$ we get the line between P_0 and P_2. For $r=\frac{1}{2}$ we get the parabola $u^2P_0 + 2u(1-u)P_1 + (1-u)^2P_2$ which passes through P_0 and P_2.

For $r > \frac{1}{2}$, we obtain hyperbolas, and for $r < \frac{1}{2}$, we obtain ellipses. Thus, we can use NURBSs to obtain both parametric polynomial curves and surfaces, and to obtain quadric surfaces.

11.17 We can write the Hermite surface as

$$\mathbf{p}(u, v) = \mathbf{u}^T \mathbf{M}_H \mathbf{Q} \mathbf{M}_H^T \mathbf{v} = \mathbf{u}^T \mathbf{A} \mathbf{v},$$

where $\mathbf{Q}$ contains the control point data and $\mathbf{M}_H$ is the Hermite geometry matrix. If evaluate $\mathbf{p}$, $\frac{\partial \mathbf{p}}{\partial u}$, $\frac{\partial \mathbf{p}}{\partial v}$, and $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$ at the corners we find that the 16 values in the matrix $\mathbf{A}$ are the 4 values at the 4 corners of the patch, the first partial derivatives $\frac{\partial \mathbf{p}}{\partial v}$ and $\frac{\partial \mathbf{p}}{\partial u}$ at the corners and the first mixed partial derivative $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$ at the corners

11.19 This process creates a quadric curve which interpolates P_0 and P_2 and lies in the triangle defined by P_0, P_1, and P_2

11.21 Nothing unusual happens other than the slope at $u = 0$ must be zero as long as the control points are still separated in parameter space.

11.25 The columns of the matrix $\mathbf{M_R}$ contain the coefficients of the blending polynomials which are

$$p_0(u) = -u^3 + 2u^2 - u,$$

$$p_1(u) = 2u^3 - 5u^2 + 2,$$

$$p_2(u) = -3u^3 + 4u^2 - u,$$

$$u^3 - u^2.$$

Note that the third and fourth polynomials can be obtained from the first and second by substituting $1 - u$ for u. We zeros of the fourth polynomial are 0, 0, and 1 so the zeros of the first are 0, 1, and 1. We can obtain the zeros of the third by factoring out u which gives a zero at 0 and solving the resulting quadratic equation to find the zeros at $\frac{-3 \pm \sqrt{7}}{2}$. The zeros of the second polynomial are thus 1 and $\frac{1 \pm \sqrt{7}}{2}$.